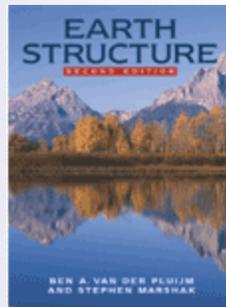


# Lecture 6

## Brittle Deformation



Earth Structure (2<sup>nd</sup> Edition), 2004  
W.W. Norton & Co, New York  
Slide show by Ben van der Pluijm

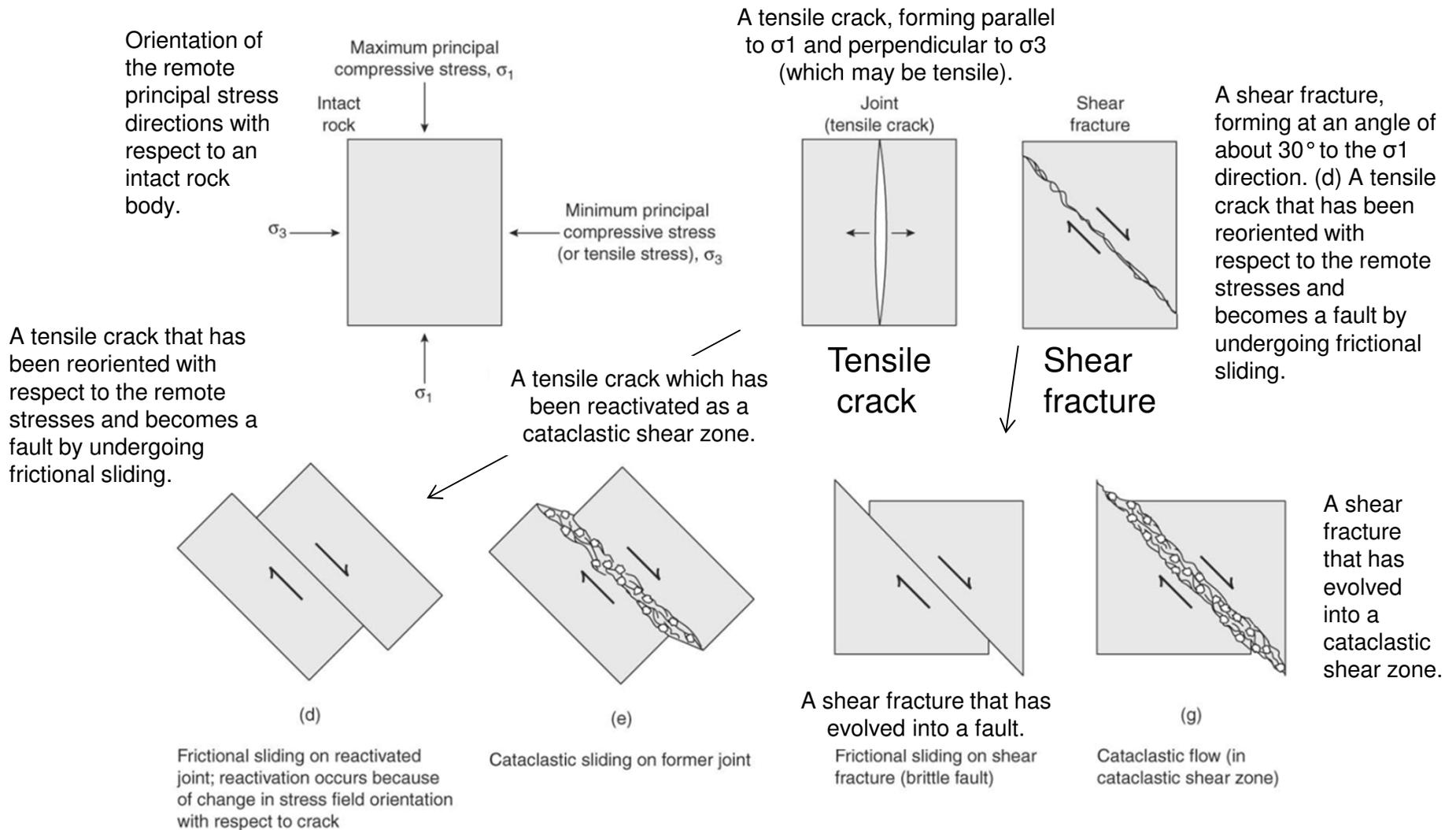
© WW Norton, unless noted otherwise

# Brittle deformation



# Types of brittle deformation Figure 6.4

**Brittle deformation** is the permanent change that occurs in a solid material due to the growth of fractures and/or due to sliding on fractures once they have formed.



# Brittle Deformation Processes

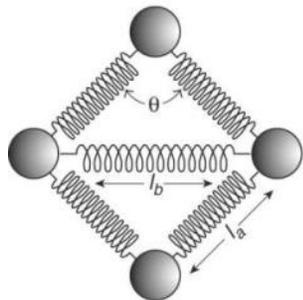
TABLE 6.2	CATEGORIES OF BRITTLE DEFORMATION PROCESSES
Cataclastic flow	This type of brittle deformation refers to macroscopic ductile flow as a result of grain-scale fracturing and frictional sliding distributed over a band of finite width.
Frictional sliding	This process refers to the occurrence of sliding on a preexisting fracture surface, without the significant involvement of plastic deformation mechanisms.
Shear rupture	This type of brittle deformation results in the initiation of a macroscopic shear fracture at an acute angle to the maximum principal stress when a rock is subjected to a triaxial compressive stress. Shear rupturing involves growth and linkage of microcracks.
Tensile cracking	This type of brittle deformation involves propagation of cracks into previously unfractured material when a rock is subjected to a tensile stress. If the stress field is homogeneous, tensile cracks propagate in their own plane and are perpendicular to the least principal stress ( $\sigma_3$ ).

# Atomic perspective

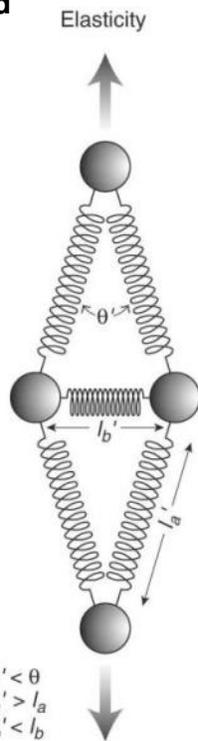
FIGURE 6.3 A sketch illustrating what is meant by stretching and breaking of atomic bonds.

The chemical bonds are represented by springs and the atoms by spheres.

Four atoms arranged in a lattice at equilibrium.



$l_a$  = undistorted length  
 $l_a'$  = distorted length

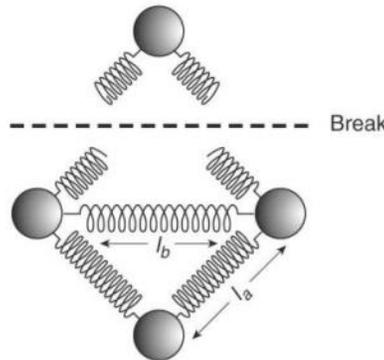


$\theta' < \theta$   
 $l_a' > l_a$   
 $l_b' < l_b$

(a)

As a consequence of stretching of the lattice, some bonds stretch and some shorten, and the angle between pairs of bonds changes.

Fracture  
 If the bonds are stretched too far, they break, and elastic strain is released



(c)

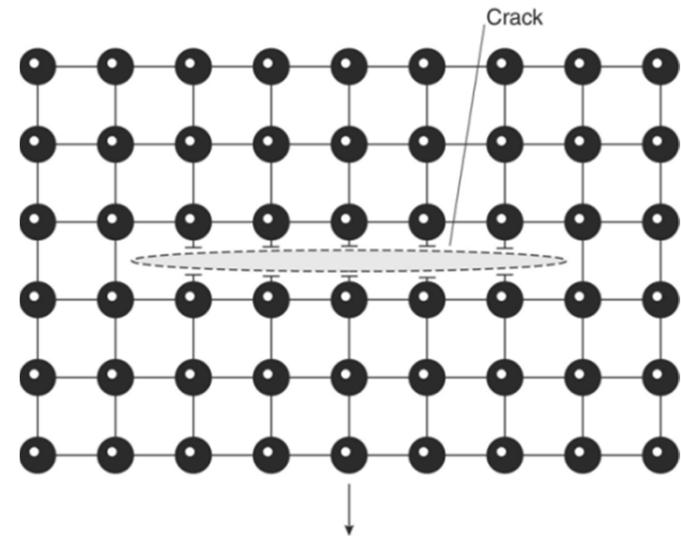


FIGURE 6.5 A cross-sectional sketch of a crystal lattice (balls are atoms and sticks are bonds) in which there is a crack. The crack is a plane of finite extent across which all atomic bonds are broken.

**Strength Paradox:** Rocks allow only few % elastic strain before ductile or brittle deformation.

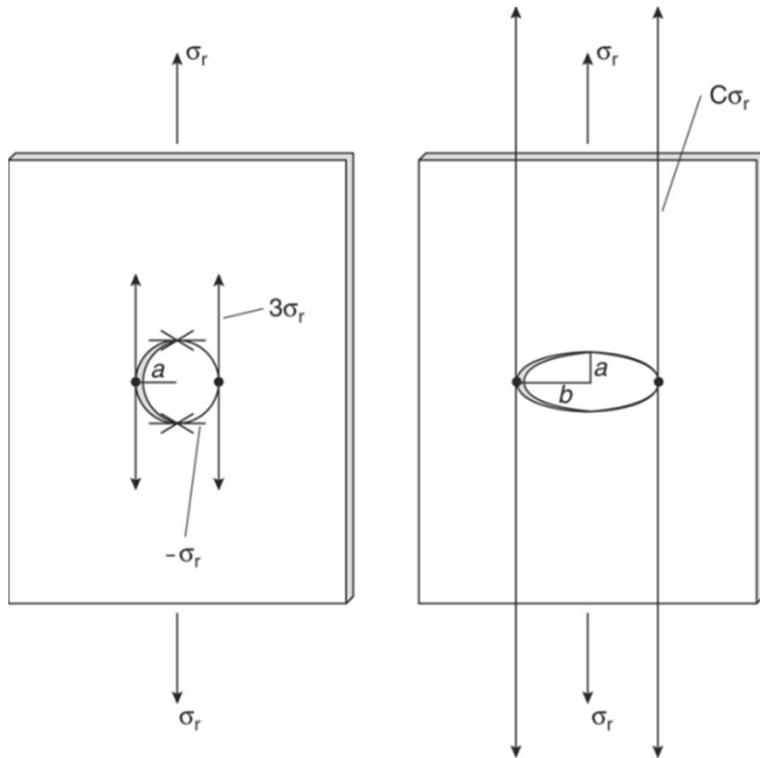
$$\sigma = E \cdot \epsilon = 10^{11} \cdot 0.1 = 10^{10} \text{ Pa}$$

Thus, theoretical strength is 1000's Mpa

Practical strength 10's MPa

# Tensile cracking Figures 6.6 and 6.7

Stress concentration adjacent to a hole in an elastic sheet. If the sheet is subjected to a remote tensile stress at its ends ( $\sigma_r$ ), then stress magnitudes at the sides of the holes are equal to  $C\sigma_r$ , where the **stress concentration factor (C)**, is  $(2b/a) + 1$ .



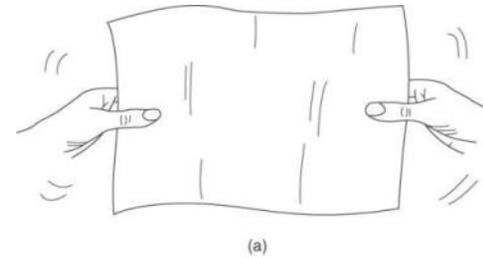
For a circular hole,  $C = 3$ .

For an elliptical hole,  $C > 3$ .

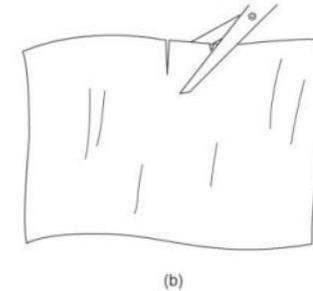
*Remote and local stress: stress concentration, C, is  $(2b/a) + 1$*

Crack  $1 \times .02 \mu\text{m}$ :  $C = 100$  ! - C becomes larger as cracks grow (larger is weaker) - cracks 'runaway'

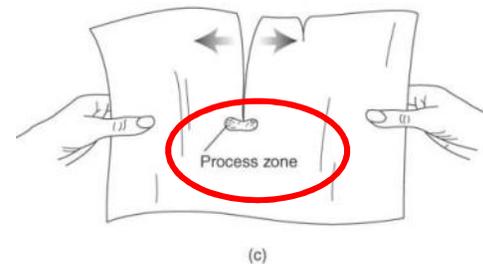
Illustration of a home experiment to observe the importance of preexisting cracks in creating stress concentrations.



An intact piece of paper is difficult to pull apart.



Two cuts, a large one and a small one, are made in the paper.



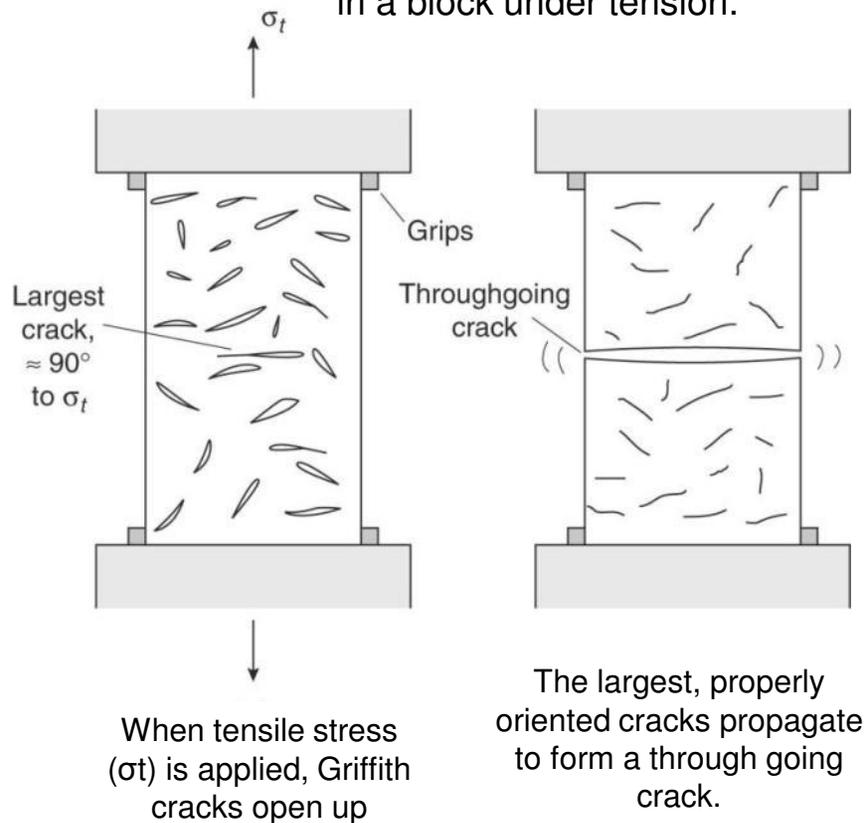
The larger preexisting cut propagates.

In the shaded area, a region called the process zone, the plastic strength of the material is exceeded and deforms.

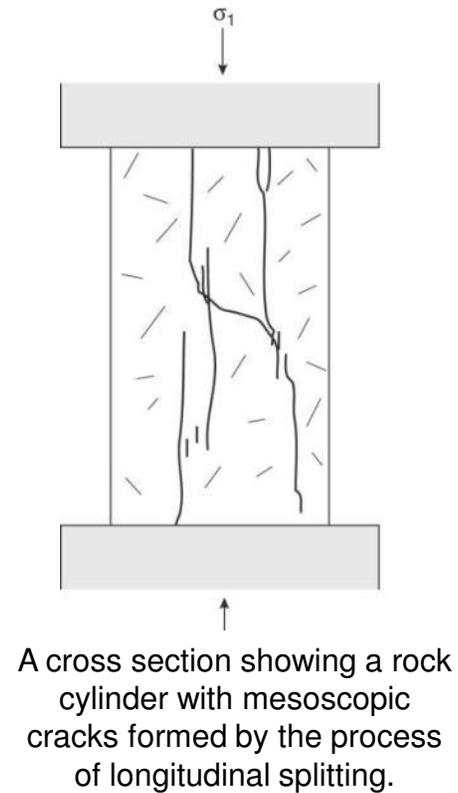
# Axial experiments: Griffith cracks Figs. 6.8 and 6.9

## Extension

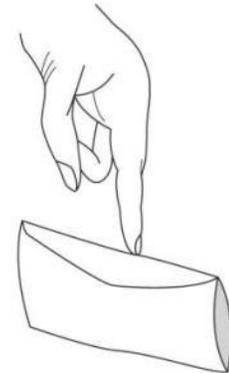
Development of a through-going crack in a block under tension.



## Compression



An "envelope" model of longitudinal splitting.



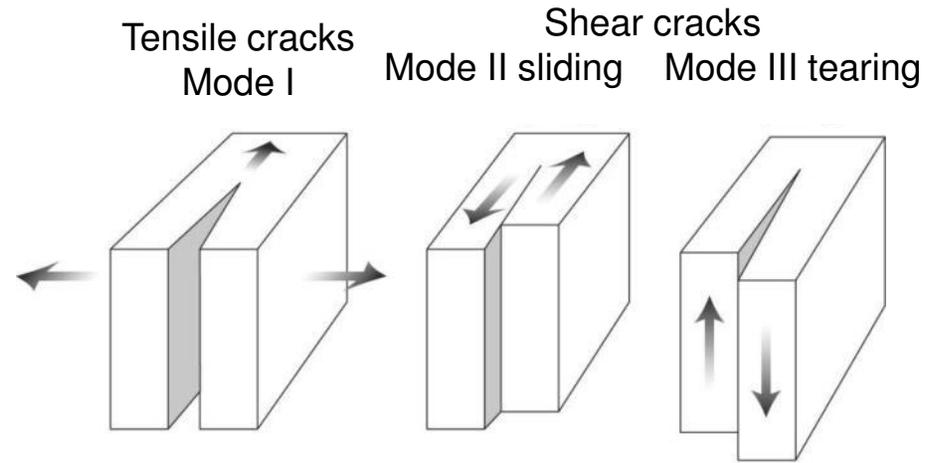
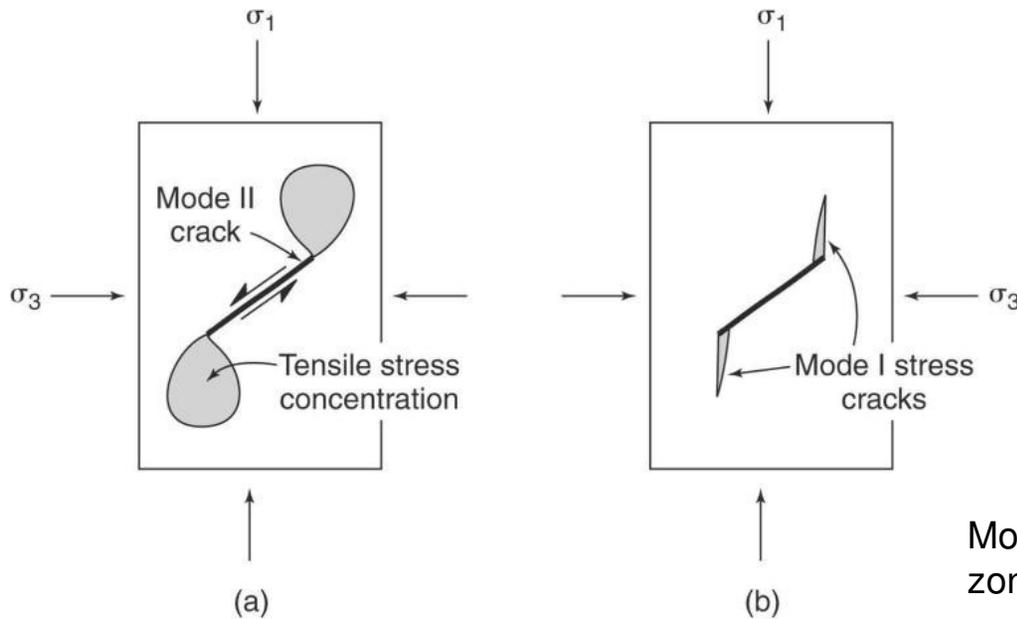
If you push down on the top of an envelope (whose ends have been cut off), the sides of the envelope will move apart.

Effect of preexisting (or Griffith) cracks: preferred activation

# Reminder: crack modes Figs. 6.11 and 6.12

## 2 Propagating shear-mode cracks and the formation of wing cracks.

A tensile stress concentration occurs at the ends of a Mode II crack that is being loaded.



**Shear fracture or crack** is a surface across which a rock loses continuity when the shear stress parallel to the surface (a traction) is sufficiently large.

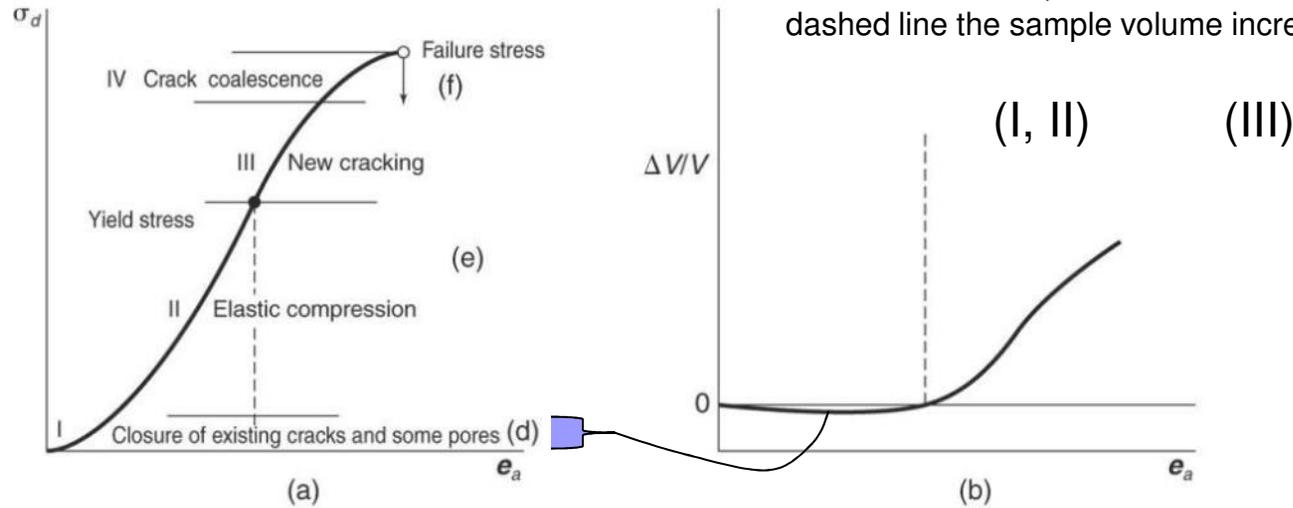
Shear cracks are **not** faults: as they propagate, they rotate into Mode I orientation ("wing cracks")

Mode I wing cracks form in the zones of tensile-crack concentration.

# Formation of shear fractures Fig. 6.14

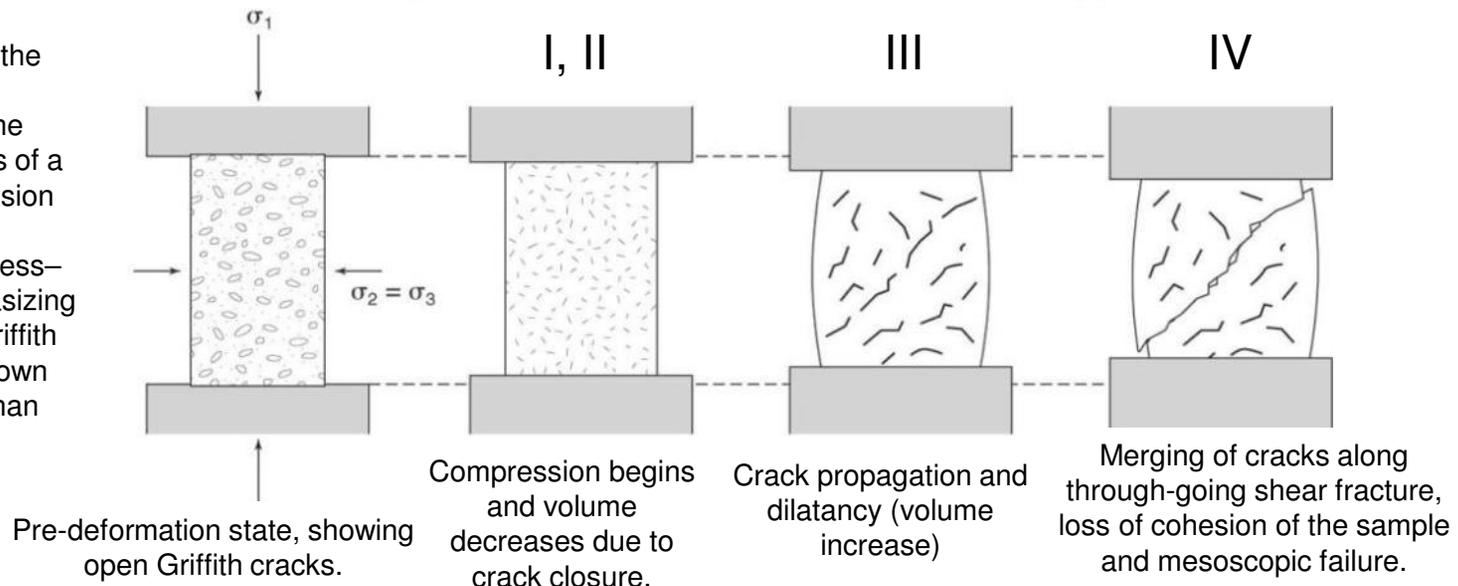
Stress–strain plot (differential stress versus axial shortening) showing the stages (I–IV) in a confined Compression experiment.

The labels indicate the process that accounts for the slope of the curve.



The changes in volume accompanying the axial shortening illustrate the phenomenon of **dilatancy**; left of the dashed line, the sample volume decreases, whereas to the right of the dashed line the sample volume increases.

Schematic cross sections showing the behavior of rock cylinders during the successive stages of a confined compression experiment and accompanying stress–strain plot, emphasizing the behavior of Griffith cracks (cracks shown are much larger than real dimensions).



# Shear Failure Criteria – 1 Fig. 6.15

Coulomb failure criterion:

$$\sigma_s = C + \mu\sigma_n$$

$\sigma_n$  is the normal stress across shear fracture at instant of failure

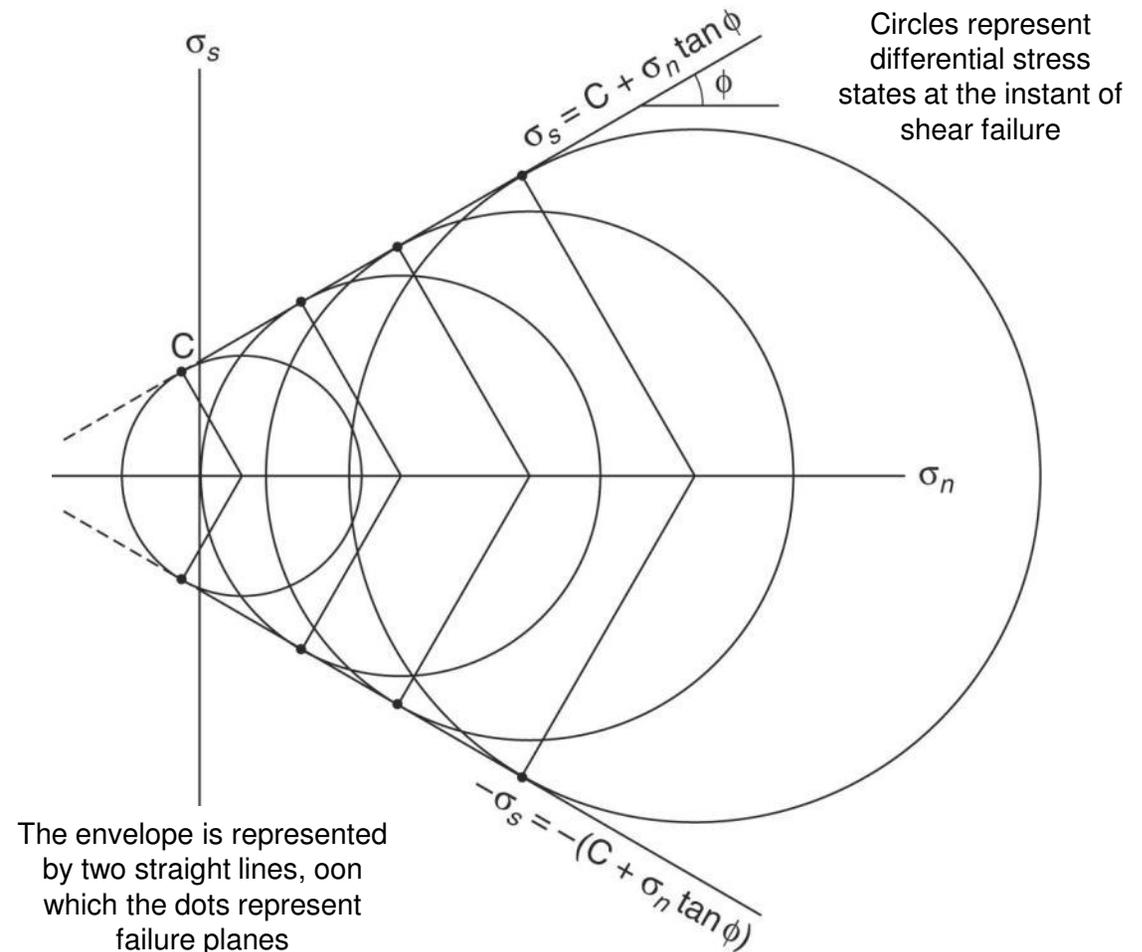
$\sigma_s$  is the shear stress parallel to fracture surface at failure

$C$  is **cohesion**, a constant that specifies shear stress necessary to cause failure if normal stress across potential fracture plane equals zero

$\mu$  is a constant, known as **coefficient of internal friction**

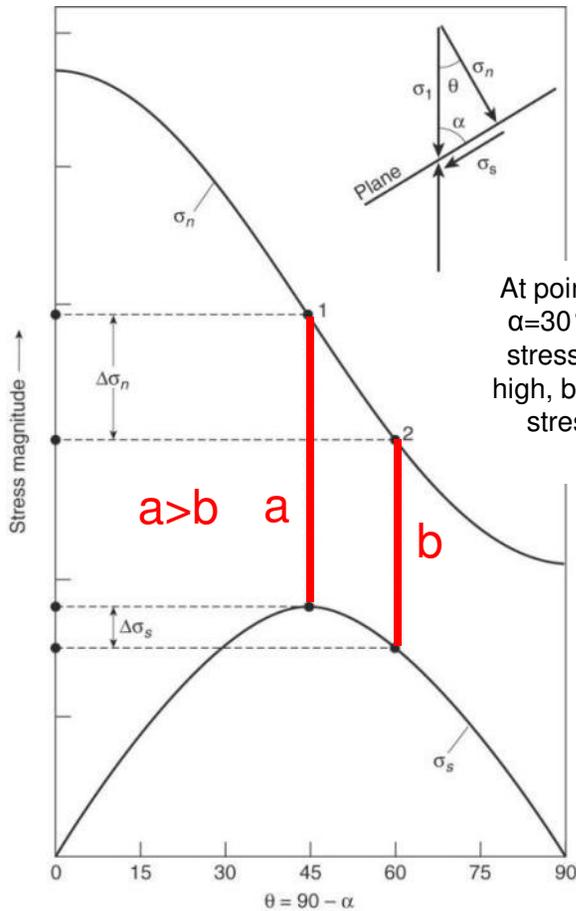
Fracture surfaces (2 !) at  $\sim 30^\circ$  to  $\sigma_1$

Mohr diagram showing Coulomb failure envelope based on a set of experiments with increasing differential stress



# Why 30° instead of 45° fracture angle with $\sigma_1$ ? Figs. 6.16 and 6.17

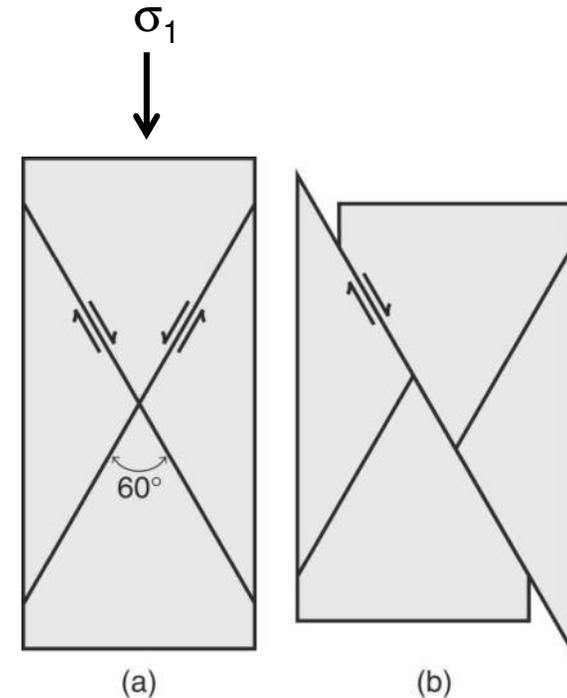
At point 1 ( $\alpha = \theta = 45^\circ$ ), shear stress is a maximum, but the normal stress across the plane is quite large.



45° is maximum shear stress, but fractures form 30° from  $\sigma_1$

At point 2 ( $\theta = 60^\circ$ ,  $\alpha=30^\circ$ ), the shear stress is still quite high, but the normal stress is much lower.

The change in magnitudes of the normal and shear components of stress acting on a plane as a function of the angle  $\alpha$  between the plane and the  $\sigma_1$  direction; the angle  $\theta = 90 - \alpha$  is plotted for comparison with other diagrams.



Cross-sectional sketch showing how only one of a pair of conjugate shear fractures (a) evolves into a fault with measurable displacement (b).

## Shear failure criteria – 2 Fig. 6.18

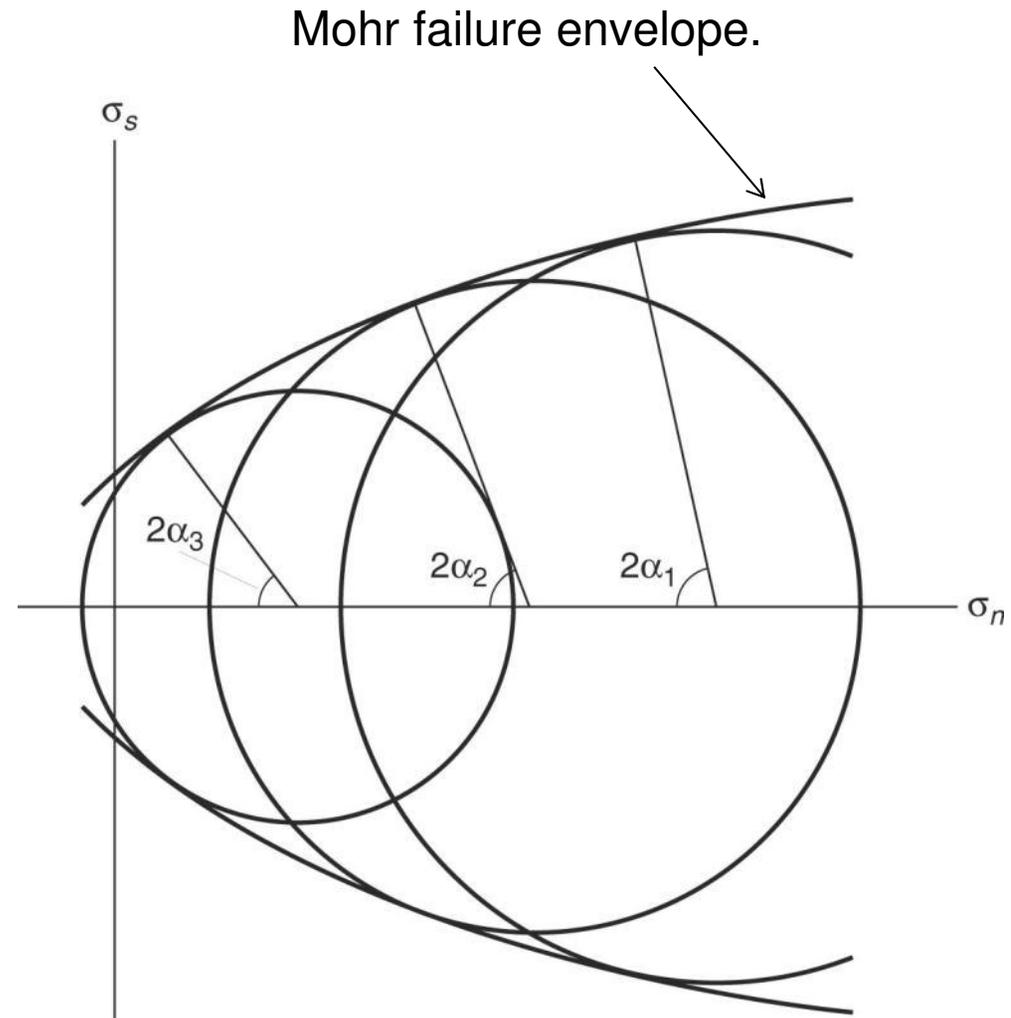
### Parabolic failure envelope:

steeper near tensile field and shallower at high  $\sigma_n$

Therefore, the value of  $\alpha$  (the angle between fault and  $\sigma_1$ ) is not constant (compare  $2\alpha_1$ ,  $2\alpha_2$ , and  $2\alpha_3$ ).

Fracture angle varies around  $30^\circ$

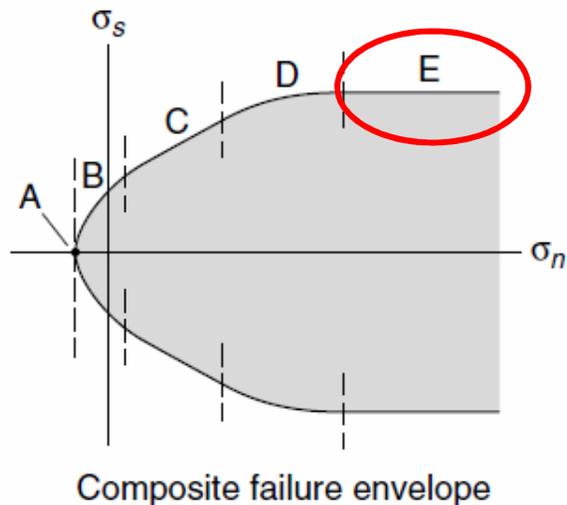
Mohr-Coulomb criterion



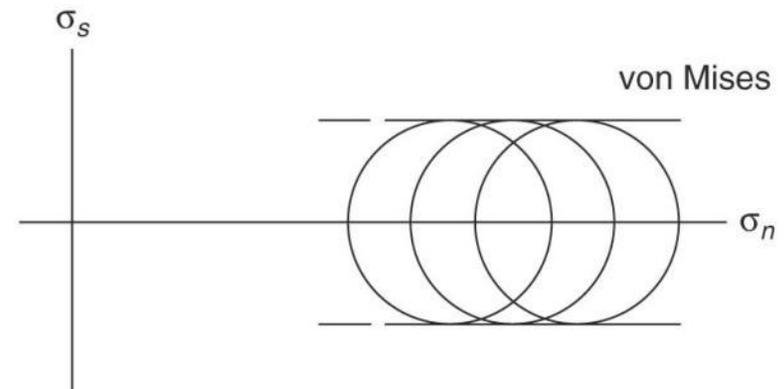
## Shear failure criteria - 3

Ductile deformation at high stress, shear stress independent  
(technically a plastic failure criterion)

- A: Tensile failure criterion
- B: Mohr (parabolic) failure criterion
- C: Coulomb (straight-line) failure criterion
- D: Brittle-plastic transition
- E: von Mises plastic yield criterion



Mohr diagram illustrating the  
*Von Mises yield criterion*



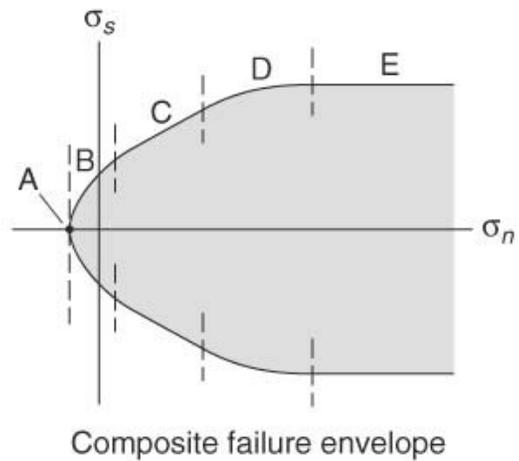
Note that the criterion is represented by two lines that parallel the  $\sigma_n$ -axis.

# Composite Failure Envelope Fig. 6.21

Tensile crack: Griffith criterion

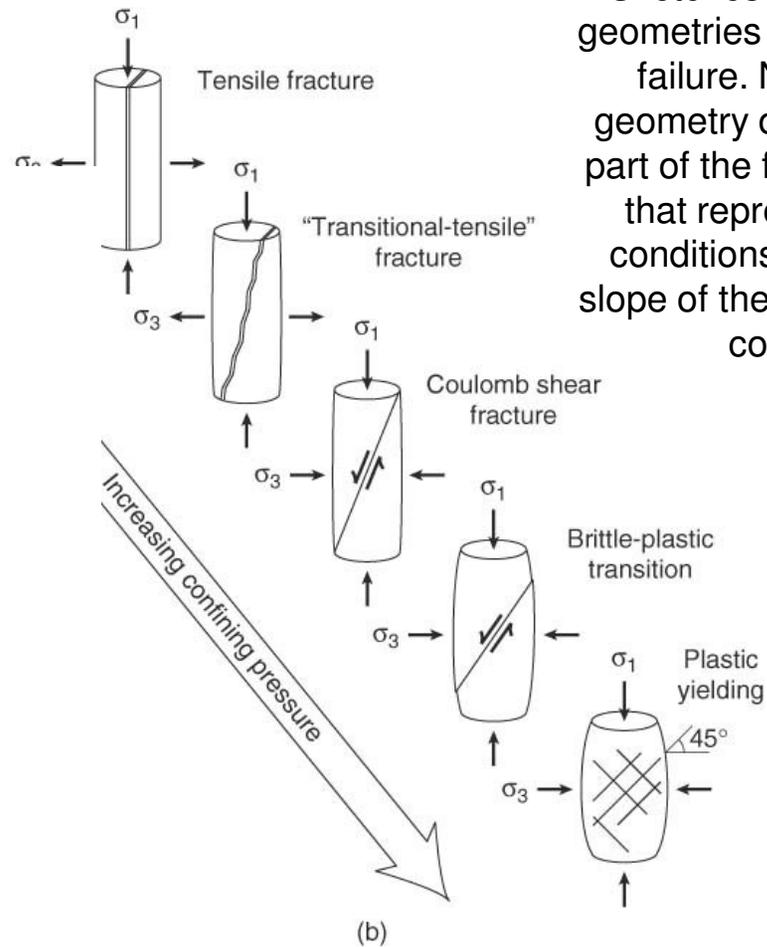
Shear fracture: Mohr-Coulomb criterion

A representative composite failure envelope on a Mohr diagram.



- A: Tensile failure criterion
- B: Mohr (parabolic) failure criterion
- C: Coulomb (straight-line) failure criterion
- D: Brittle-plastic transition
- E: von Mises plastic yield criterion

(a)



Sketches of the fracture geometries that form during failure. Note that the geometry depends on the part of the failure envelope that represents failure conditions, because the slope of the envelope is not constant.

# Fault Types





# Anderson's Theory of Faulting

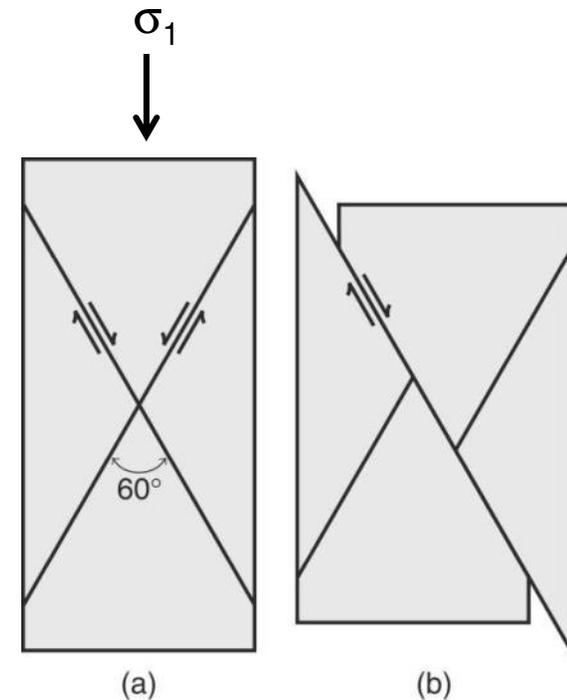
# Anderson's Theory of Faulting

Faulting represents a response of rock to shear stress, so it only occurs when the differential stress ( $\sigma_d = \sigma_1 - \sigma_3 = 2\sigma_s$ ) does not equal zero.

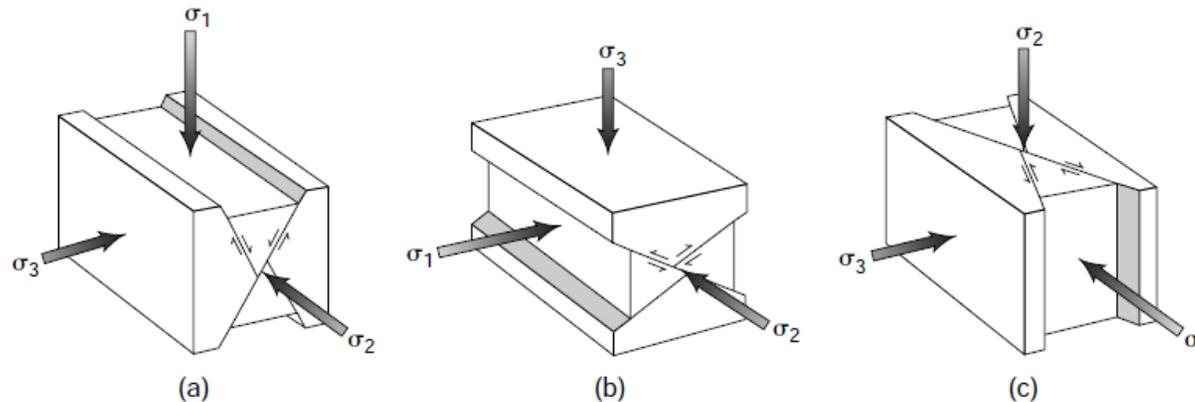
Because shear-stress magnitude on a plane changes as a function of the orientation of the plane with respect to the principal stresses, we should expect a relationship between the orientation of faults formed during a tectonic event and the trajectories of principal stresses during that event.

Indeed, faults that initiate as Coulomb shear fractures will form at an angle of about  $30^\circ$  to the  $\sigma_1$  direction and contain the  $\sigma_2$  direction.

This relationship is called **Anderson's theory of faulting**.



**Anderson's Theory of Faulting** states that in the Earth-surface reference frame, normal faulting occurs where  $\sigma_2$  and  $\sigma_3$  are horizontal and  $\sigma_1$  is vertical, thrust faulting occurs where  $\sigma_1$  and  $\sigma_2$  are horizontal and  $\sigma_3$  is vertical, and strike-slip faulting occurs where  $\sigma_1$  and  $\sigma_3$  are horizontal and  $\sigma_2$  is vertical



**FIGURE 8.27** Anderson's theory of faulting predicts (a) [high-angle] normal faults, (b) [low-angle] reverse faults [or thrusts], and (c) [vertical] strike-slip faults.

- Recall the role of the normal stress, where the ratio of shear stress to normal stress on planes orientated at about  $30^\circ$  to  $\sigma_1$  is at a maximum. The Earth's surface is a "free surface" (the contact between ground and air/fluid) that cannot, therefore, transmit a shear stress.
- Therefore, regional principal stresses are parallel or perpendicular to the surface of the Earth in the upper crust.
- Considering that gravitational body force is a major contributor to the stress state, and that this force acts vertically, stress trajectories in homogeneous, isotropic crust can maintain this geometry at depth.

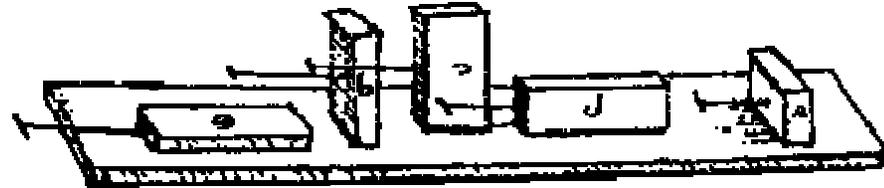
- 
- Moreover, the dip of thrust faults should be  $\sim 30^\circ$ , the dip of normal faults should be  $\sim 60^\circ$ , and the dip of strike-slip faults should be about vertical.
  - For example, if the  $\sigma_1$  orientation at convergent margins is horizontal, Anderson's theory predicts that thrust faults should form in this environment, and indeed belts of thrust faults form in collisional mountain belts.
  - Anderson's theory is a powerful tool for regional analysis, but we cannot use this theory to predict all fault geometries in the Earth's crust for several reasons.
  - First, faults do not necessarily initiate in intact rock.
  - The frictional sliding strength of a preexisting surface is less than the shear failure strength of intact rock; thus, preexisting joint surfaces or faults may be reactivated before new faults initiate, even if the preexisting surfaces are not inclined at  $30^\circ$  to  $\sigma_1$  and do not contain the  $\sigma_2$  trajectory.
  - Preexisting fractures that are not ideally oriented with respect to the principal stresses become oblique-slip faults.

Second, a fault surface is a material feature in a rock body whose orientation may change as the rock body containing the fault undergoes progressive deformation.

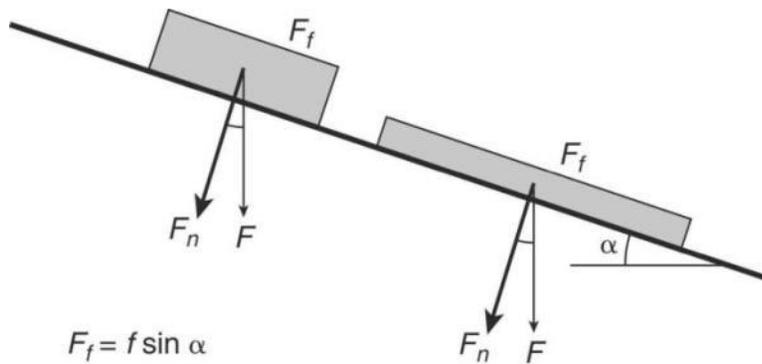
Thus, the fault may rotate into an orientation not predicted by Anderson's theory

**Frictional sliding** refers to movement on a surface that takes place when shear stress parallel to surface exceeds frictional resistance to sliding.

FIGURE 6.22 Frictional sliding of objects with same mass, but with different (apparent) contact areas.



(15<sup>th</sup> C da Vinci experiments)



$$F_f = f \sin \alpha$$

$$F_n = f \cos \alpha$$

$$F_f / F_n = \sin \alpha / \cos \alpha = \mu$$

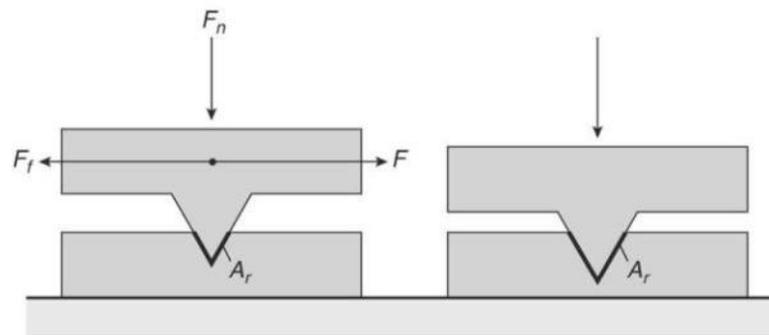
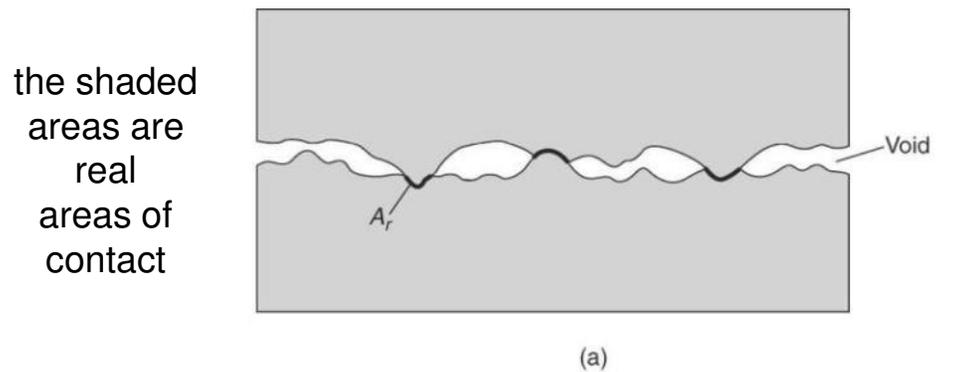
The friction coefficients and, therefore, sliding forces ( $F_f$ ) are equal for both objects, regardless of (apparent) contact area.

Amonton's Laws of Friction:

- Frictional force is a function of normal force.
- Frictional force is independent of (apparent) area of contact.
- Frictional force is (mostly) independent of material used.

# Concepts of Asperities Fig. 6.23

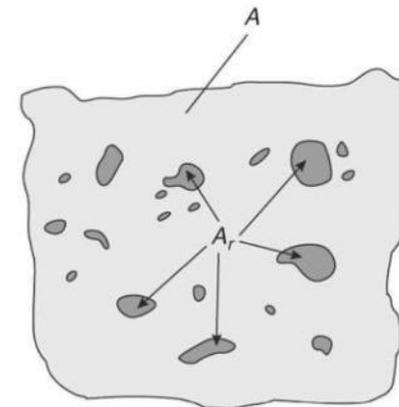
Schematic cross-sectional close-up showing the irregularity of a fracture surface and the presence of voids and asperities along the surface



Idealized asperity showing the consequence of changing the load (normal force) on the real area of contact

The bumps and irregularities that protrude from a (rough) surface are called **asperities**.

$A$  = (apparent) area  
 $A_r$  = real area

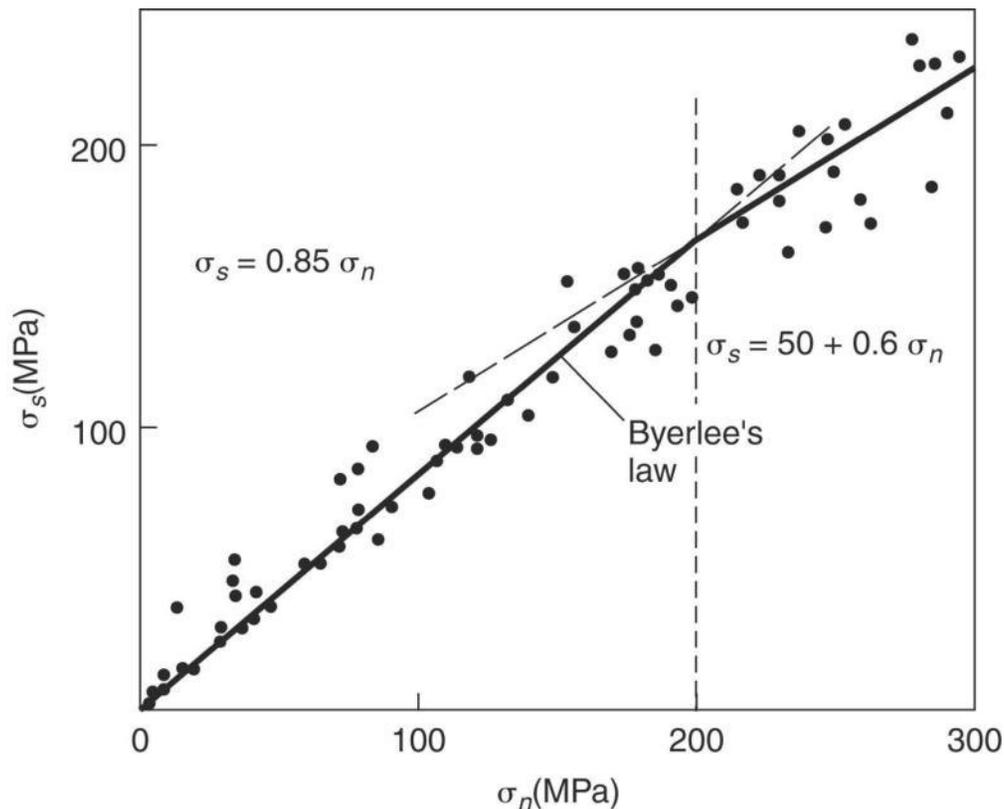


Map of a fracture surface

## Frictional Sliding Criteria (Byerlee's Law) Fig. 6.24

Graph of shear stress and normal stress values at the initiation of sliding on preexisting fractures in a variety of rock types.

The best-fit line defines Byerlee's law, which is defined for two regimes.



**Byerlee's Law** depends on  $\sigma_n$

For  $\sigma_n < 200$  MPa, the best-fitting criterion is  $\sigma_s = 0.85\sigma_n$ .

For  $200 \text{ MPa} < \sigma_n < 2000 \text{ MPa}$ , the best-fitting criterion is  $\sigma_s = 50 \text{ MPa} + 0.6\sigma_n$ .

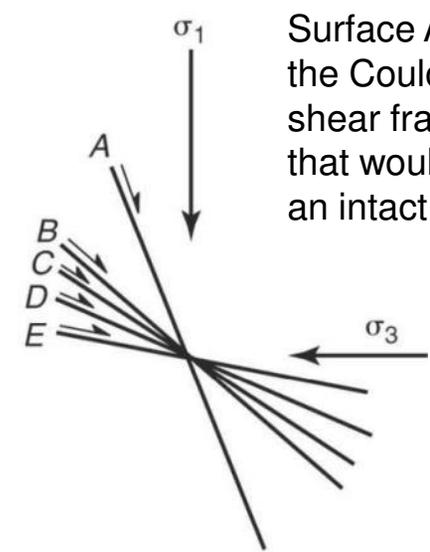
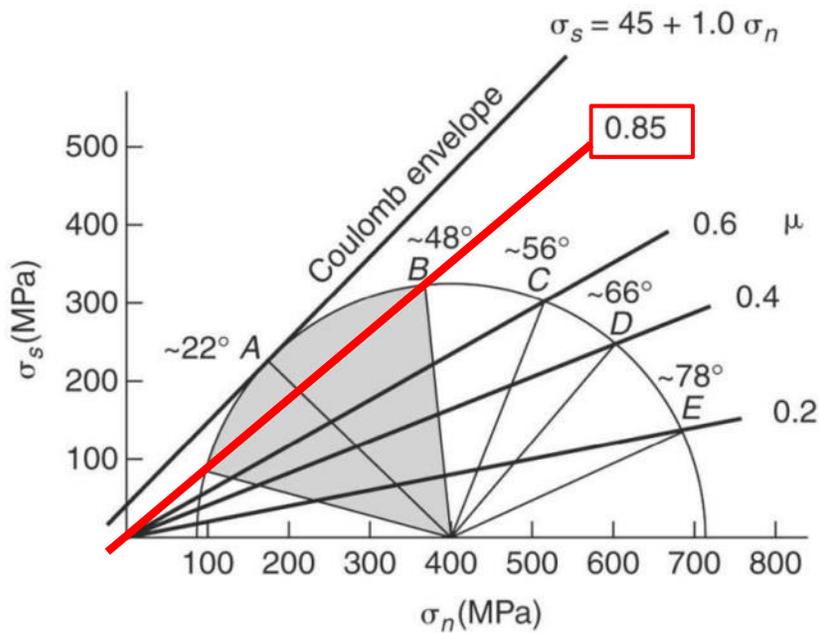
**coefficient of friction ( $\mu$ ) is a constant =  $\sigma_s / \sigma_n$**

$$\mu = 0.6-0.85 (\sim 0.7)$$

# Sliding or Fracturing? Fig. 6.25

- Mohr diagram based on experiments with Blair dolomite, showing how a single stress state (Mohr circle) would contact the frictional sliding envelope before it would contact the Coulomb envelope (heavy line).
- Sliding occurs on surfaces between intersections with the friction envelope (marked by shaded area for friction envelope  $\mu = 0.85$ ) before new fracture initiation.

Preexisting surfaces B to E are surfaces that will slide with decreasing friction coefficients.



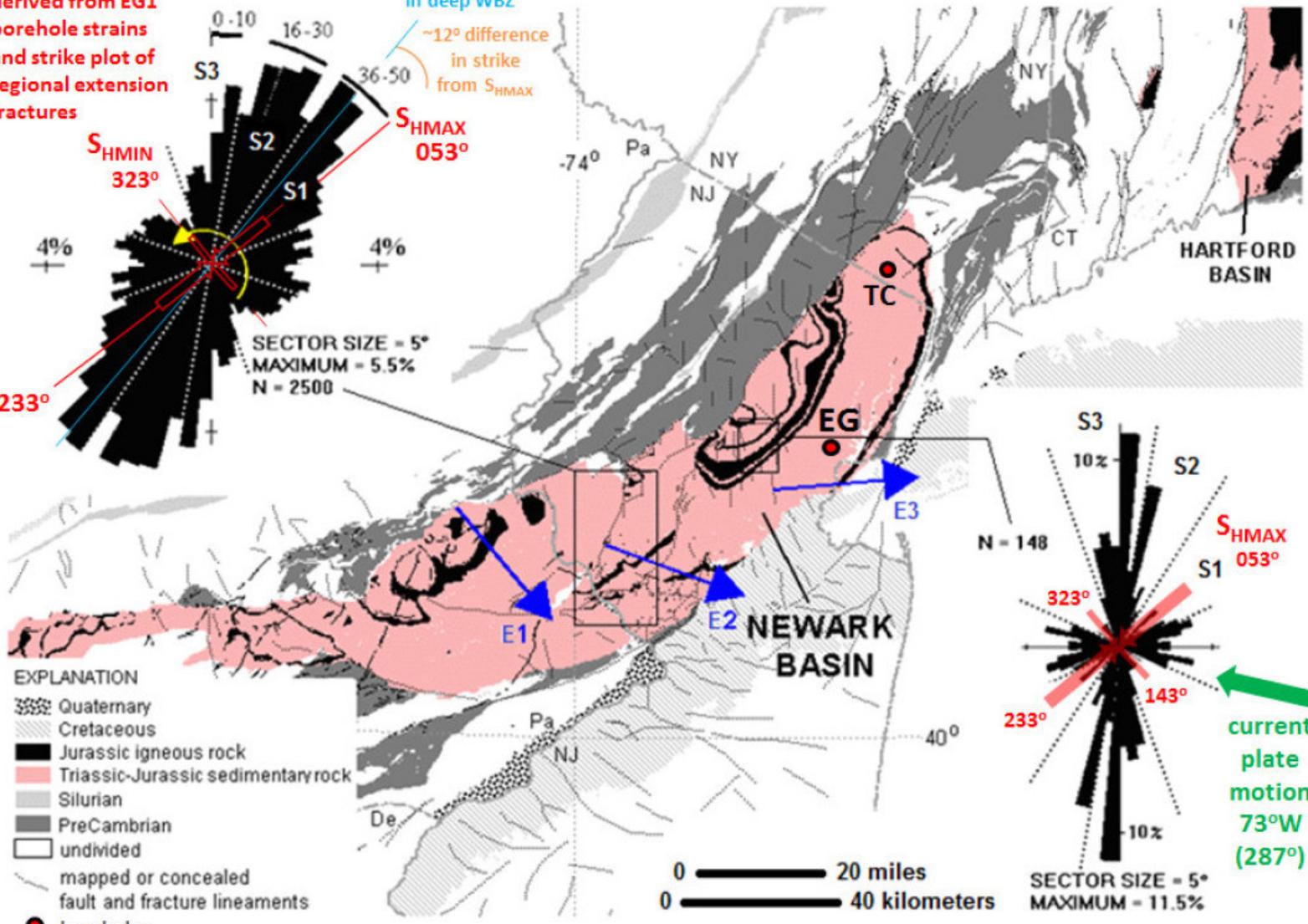
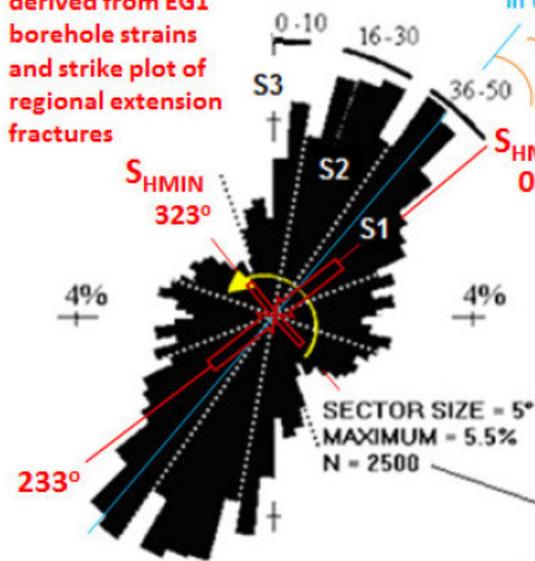
Surface A in (b) is the Coulomb shear fracture that would form in an intact rock.

- Consider the geologic relevance of decreasing friction coefficients for stress state, failure, and fracture orientation.

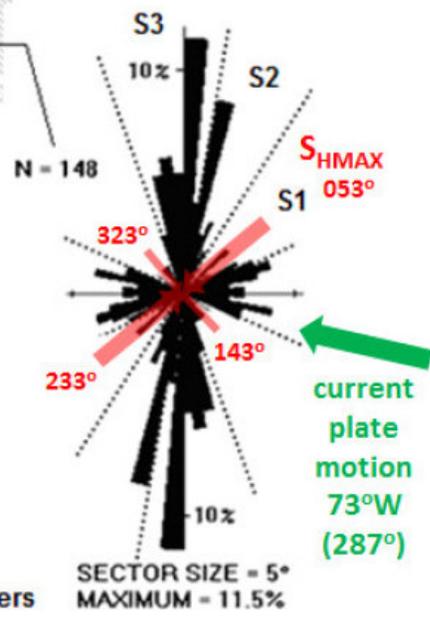
coefficient of friction ( $\mu$ ) is constant =  $\sigma_s / \sigma_n$

Crustal, in-situ, compressive, horizontal-principal-stress axes ( $S_H$ ) derived from EG1 borehole strains and strike plot of regional extension fractures

041° mean strike of open fractures in deep WBZ  
 ~12° difference in strike from  $S_{HMAX}$



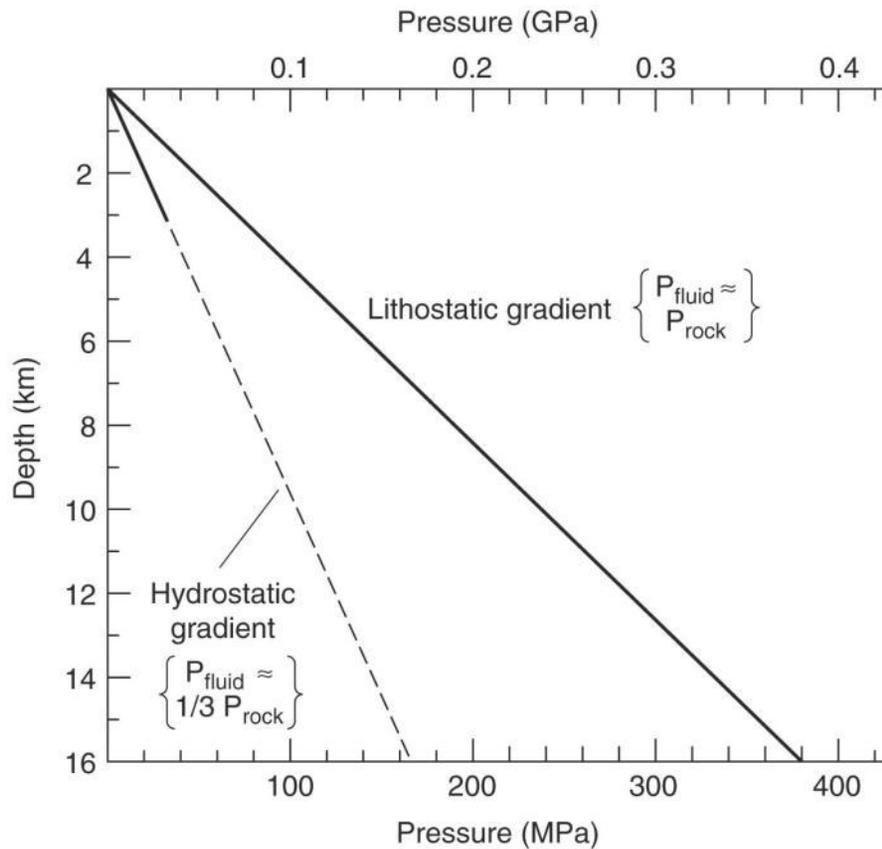
- EXPLANATION
- Quaternary
  - Cretaceous
  - Jurassic igneous rock
  - Triassic-Jurassic sedimentary rock
  - Silurian
  - PreCambrian
  - undivided
  - mapped or concealed fault and fracture lineaments
  - boreholes



RIDER GEO-310 GC Herman Rev. 09/13/2014

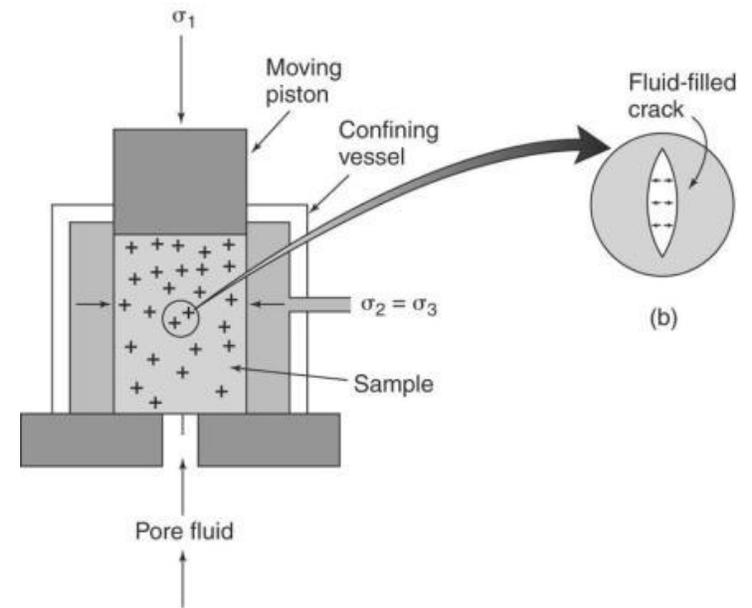
# Effects of Fluids on Tensile Crack Growth Fig. 2.26

Graph of lithostatic versus hydrostatic pressure as a function of depth in the Earth's crust.



## Lithostatic pressure

$$P_1 = \rho \cdot g \cdot h, \text{ weight of overlying column of rock } (\rho = 2500\text{--}3000 \text{ kg/m}^3).$$

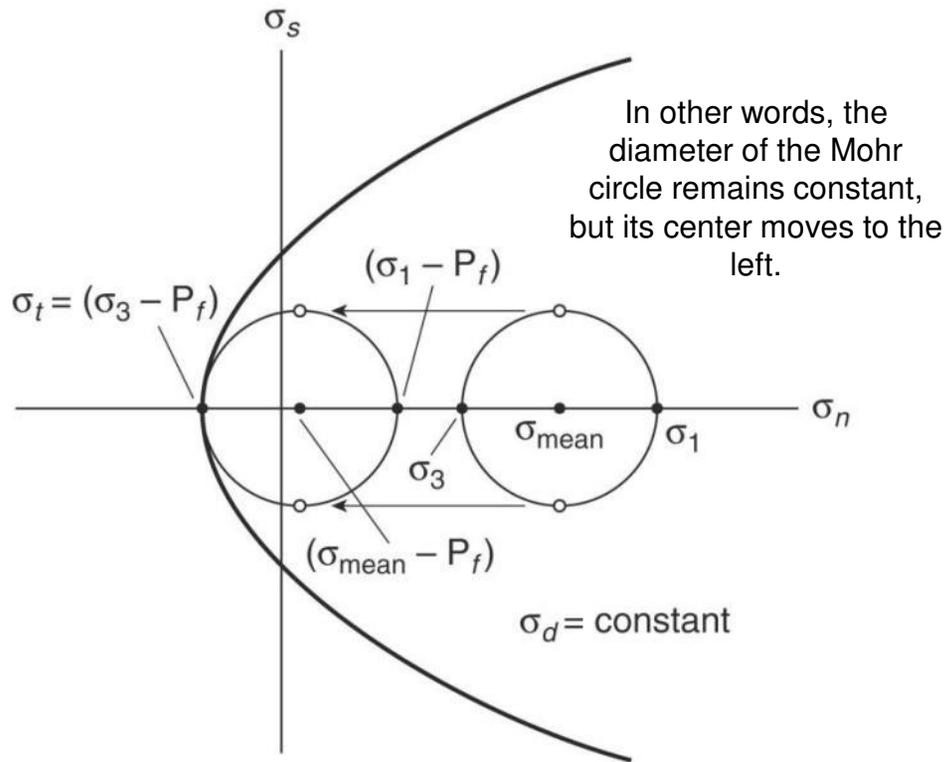


## Hydrostatic (fluid) pressure

$$P_f = \rho \cdot g \cdot h, \text{ where } \rho \text{ is density of water } (1000 \text{ kg/m}^3), g \text{ is gravitational constant } (9.8 \text{ m/s}^2), \text{ and } h \text{ is depth}$$

# Fluid Pressure and Effective Stress

Mohr diagram showing how an increase in pore pressure moves the Mohr circle toward the origin.

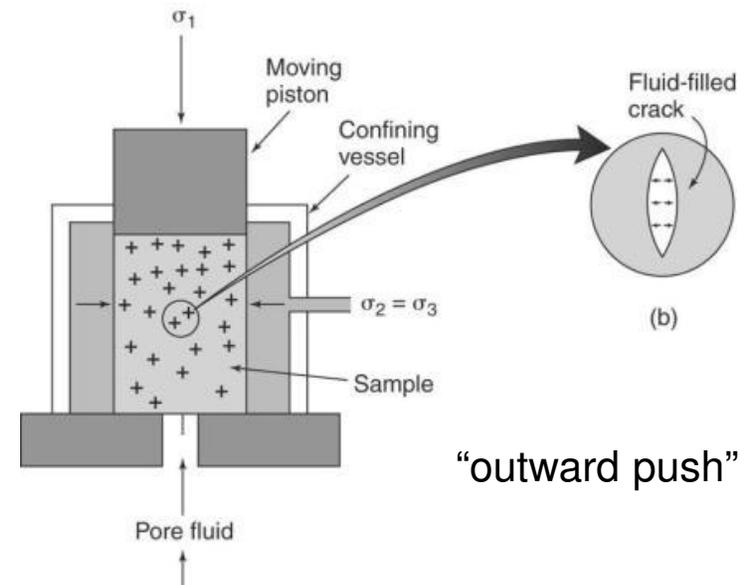


The increase in pore pressure decreases the mean stress ( $\sigma_{mean}$ ), but does not change the magnitude of differential stress ( $\sigma_1 - \sigma_3$ )

## Hydraulic fracturing

$$\sigma_s = C + \mu (\sigma_n - P_f) \text{ [fracturing]}$$

$$\sigma_s = \mu (\sigma_n - P_f) \text{ [sliding]}$$



$(\sigma_n - P_f)$  is commonly labeled  $\sigma_n$ , the effective stress.

$$\text{So, } \mu_{\text{effective}} = \mu (1 - P_f / \sigma_n)$$

$$\mu_{\text{effective}} \leq \mu$$

coefficient of friction ( $\mu$ ) is constant =  $\sigma_s / \sigma_n$

# Slip and Earthquakes Looking ahead Fig. 8.36

- The stress drops (dashed lines) correspond to slip events.
- Associated microfracturing activity is also indicated.

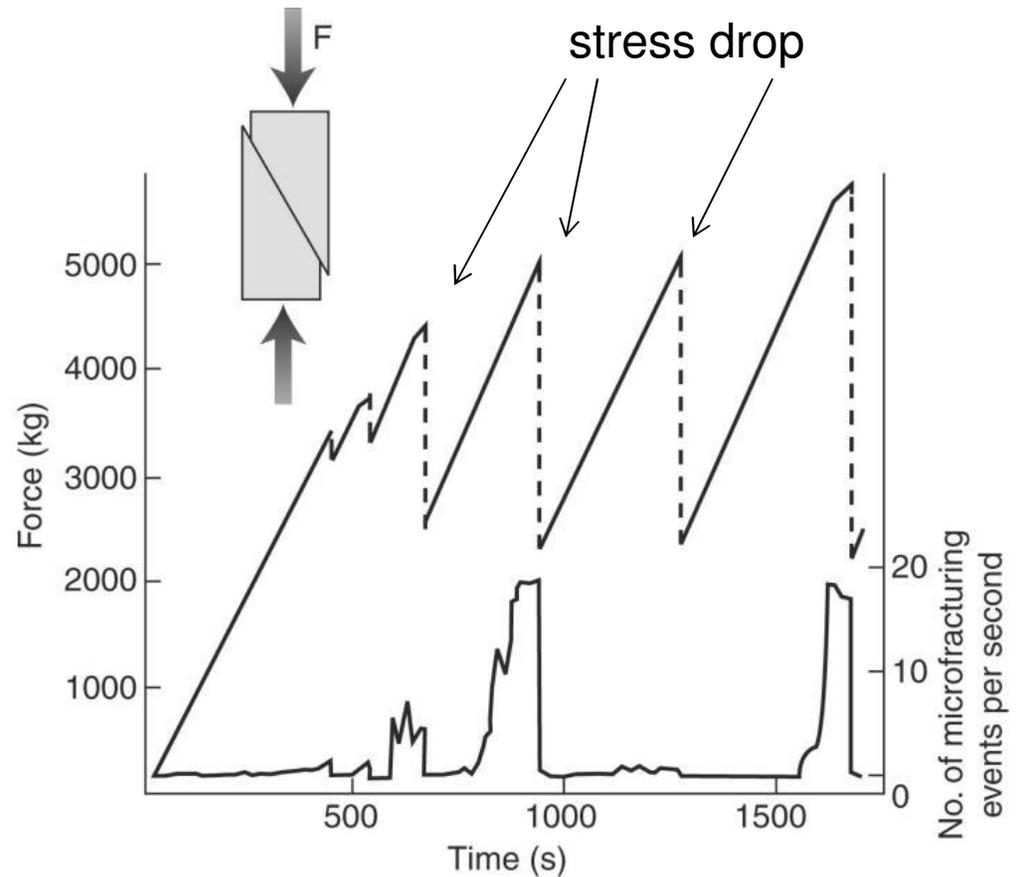
Seismic slip (earthquake)

Aseismic slip (creep)

stress build-up then partial stress release (“stress drop”).

Note: Stress drop is 1-10 MPa, i.e. 1/10<sup>th</sup> of stress state !

Laboratory frictional sliding experiment on granite, showing stick-slip behavior.



## Limiting stress conditions for sliding Fig. 8.30

Graph showing variation in differential stress necessary to initiate sliding on reverse, strike-slip, and normal faults, as a function of depth.

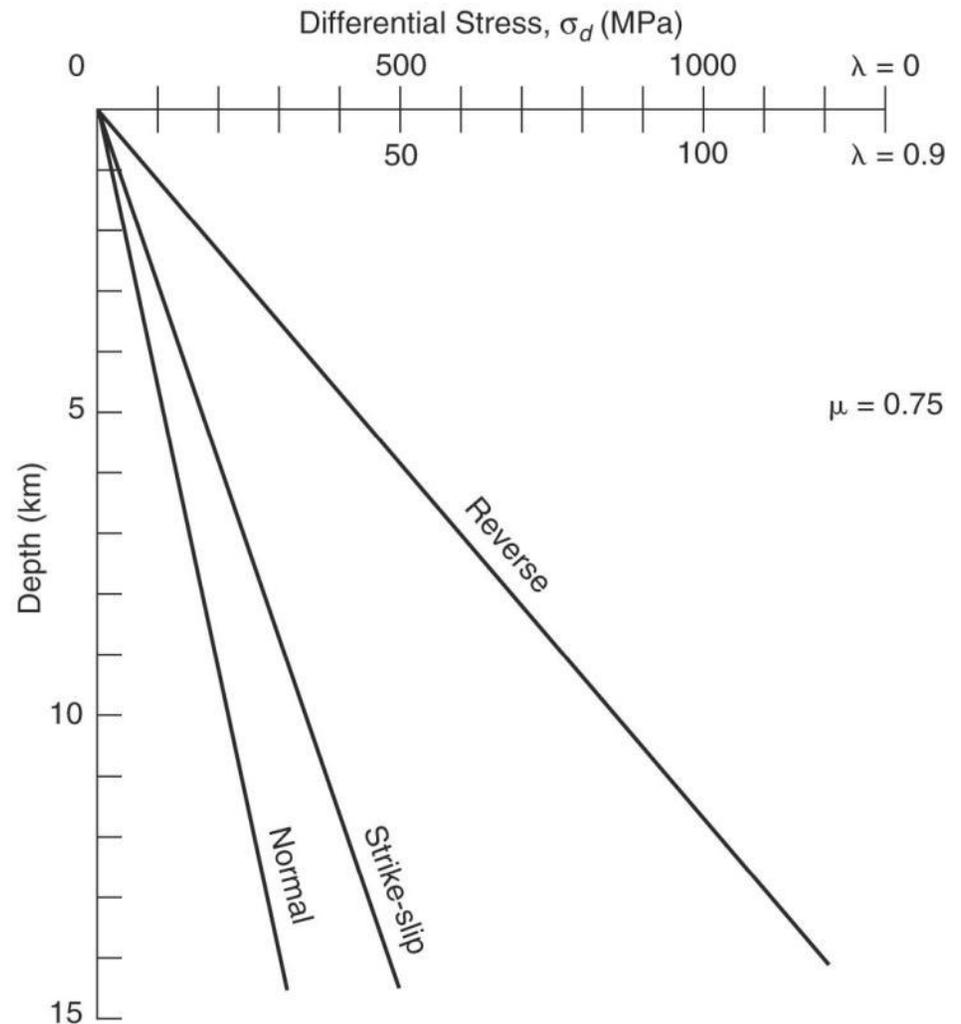
The relationship is given by Equation 8.1, assuming a friction coefficient,  $\mu=0.75$ , and a fluid pressure parameter,  $\lambda=0$  (no fluid present) and  $\lambda = 0.9$  (fluid pressure is 90% of lithostatic pressure).

$$\sigma_d \geq \beta (\rho \cdot g \cdot h) \cdot (1 - \lambda)$$

$\sigma_d$  is differential stress ( $= 2\sigma_s$ )

$\beta$  is 3, 1.2, and 0.75 for reverse, strike-slip, and normal faulting

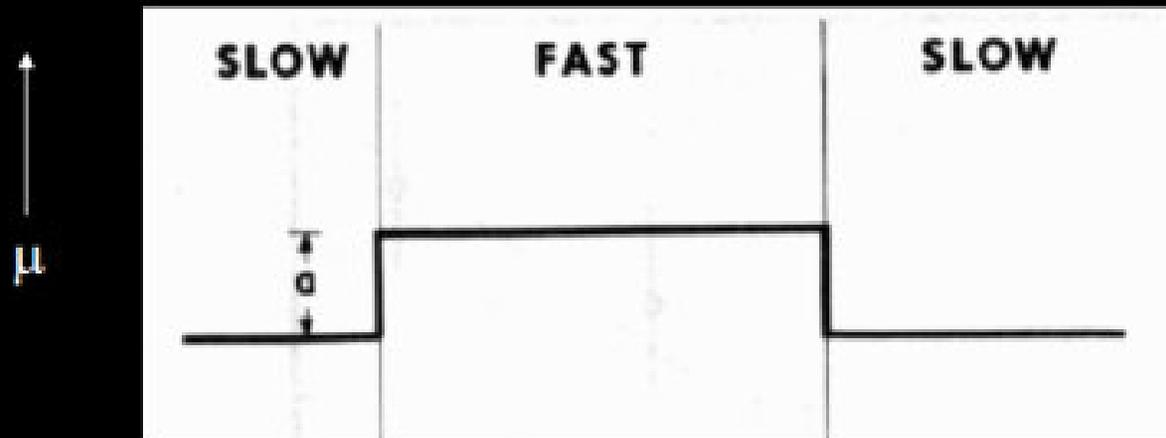
$\lambda = P_f/P_l$ , ratio of pore-fluid pressure and lithostatic pressure ( $\lambda$  ranges from  $\sim 0.4$  for hydrostatic fluid pressure to 1 for lithostatic fluid pressure)



## Rate and Friction State

If the sliding velocity,  $V$  is changed, then the resistance to sliding also changes:

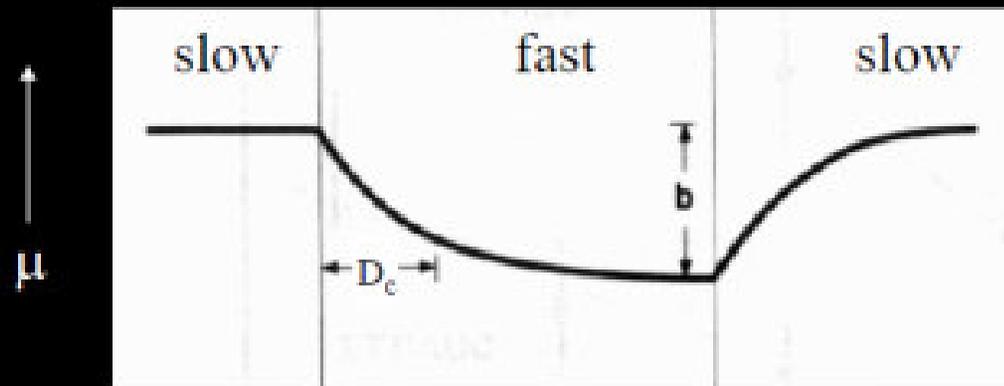
- Rate-dependent effects



## Rate and Friction State

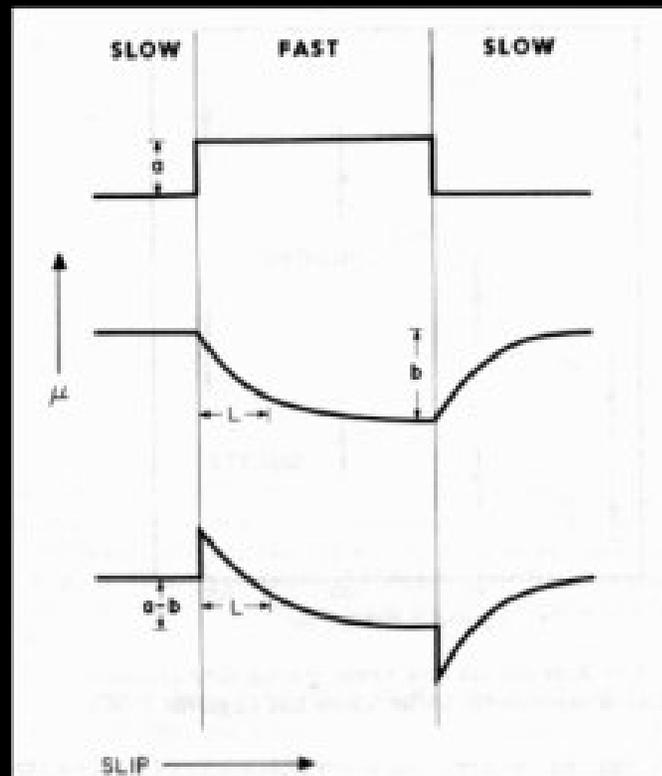
If the sliding velocity,  $V$  is changed, the sliding surface 'remembers' how strong it was, but evolves towards a new strength

- State-dependent effects



## Rate and Friction State

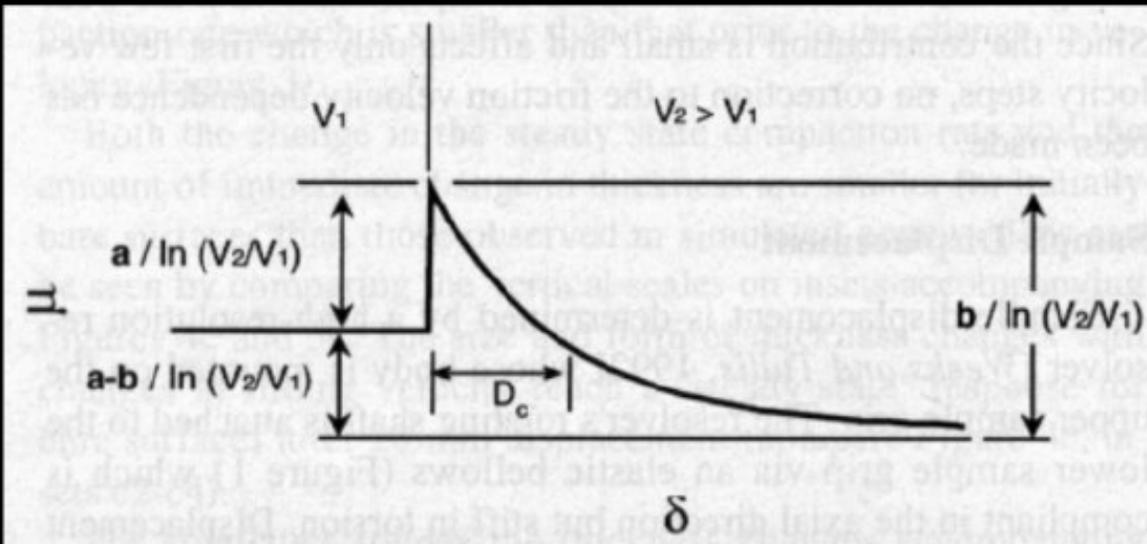
A combination of the rate- and state-dependent friction effects looks like:



Rate effect

State effect

Combined rate and state effect



**Hence:**

$a$  is related to the change in rate

$b$  is related to the change in state

$D_c$  is called the *critical slip distance*

if  $a - b < 0$ , then *unstable slip* can occur

- velocity weakening
- earthquakes

if  $a - b > 0$ , then *stable sliding* will occur

- velocity strengthening
- fault 'creep'